1. A random variable $X$ is said to be memoryless if $P\{X>s+t \mid X>t\} = P\{X>s\}$ for all $s, t \geq 0$. Show that variable $X$ is memoryless when $X$ has an exponential distribution with parameter $\lambda$; i.e., $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$. (15%)

2. Suppose the $X$ has pdf (probability density function):
   
   $$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

   Let $G(X) = 3X + 1$. Find the pdf of $Y = G(X)$. (15%)

3. A company wants to test whether a small price reduction is enough to increase sales of its product. Randomly chosen data on 15 weekly sales totals at outlets in a given area before the price reduction shows a sample mean of $6,598$ and a sample standard deviation of $844$. A random sample of 12 weekly sales totals after the small price reduction gives a sample mean of $6,870$ and a sample standard deviation of $669$. ($t_{(0.1; 25)} = 1.316, t_{(0.05; 25)} = 1.708, t_{(0.025; 25)} = 2.060$)

   (1) Is there evidence that the small price reduction is enough to increase sales of the product? ($\alpha = 0.1$) (10%)

   (2) Compute a 95% confidence interval for the difference between the two means. (10%)

4. The sales manager of a click-and-mortar company believes that Internet advertising is the most effective way to market their products. The sales manager recorded the amount of money spent on advertising and the amount of sales for 20 randomly selected months. The average cost for Internet advertising for the 20 months was $110,000. The average sale volume for the 20 months was $675,000. The following sample statistics were found from the data for the 20 months.

   $$\sum (X - \bar{X})^2 = 205.3 \quad \sum (Y - \bar{Y})^2 = 341.6 \quad \sum (X - \bar{X})(Y - \bar{Y}) = 198.4$$

   where $Y$ represents the sales volume (in thousands) and $X$ represents the Internet advertising costs (in thousands of dollars).

   (1) Calculate the least squares estimated regression line. (5%)

   (2) Compute the coefficient of determination. (5%)

   (3) Find a 95% prediction interval for the monthly sales volume if the Internet advertising expenditure during one particular month is $120,000$. (5%)

   (4) What are the necessary assumptions for the validity of the procedure in part (3). (5%)
5. Supermarket chains often carry products with their own brand labels. A supermarket conducted a taste test to determine whether there was a difference in taste among the four brands of food products it carried (A, B, C, D). A sample of 200 people surveyed, and the results of their preferences are summarized as follows:

Brand Preferences:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>39</td>
<td>57</td>
<td>55</td>
<td>49</td>
</tr>
</tbody>
</table>

Is there any evidence of a difference in preference for the four brands? Test at 5% significance level.

\( \chi^2 (0.05; 3) = 7.81, \chi^2 (0.05; 2) = 5.99 \) (15%)

6. A projector manufacturer is introducing a new product specifically targeted at the home market and wishes to compare the effectiveness of three strategies: Internet stores, home electronics stores, and department stores. The numbers of sales are shown below.

Internet store: 5, 4, 3, 3, 3
Home electronics store: 9, 7, 8, 6, 5
Department store: 7, 4, 8, 4, 3

(1) Test the hypothesis that there is no difference between the means of the retailers (\( \alpha = 0.05 \)).

(10%)

(2) Use Scheffe’s multiple comparison technique to determine which groups differ in mean sales.

(\( \alpha = 0.05 \)). (5%)

\( F (0.05; 2,12) = 3.89 \)